One-dimensional Port-and-Sweep Solitaire Armies

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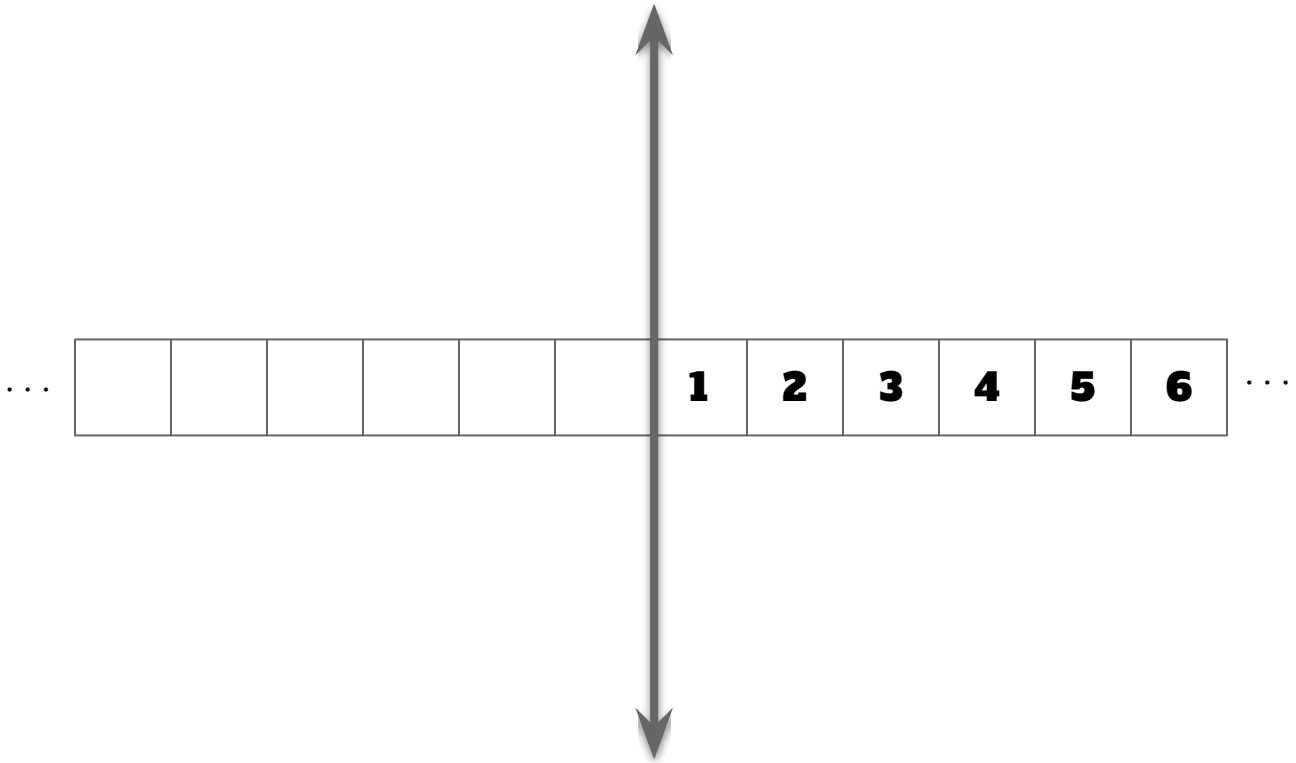
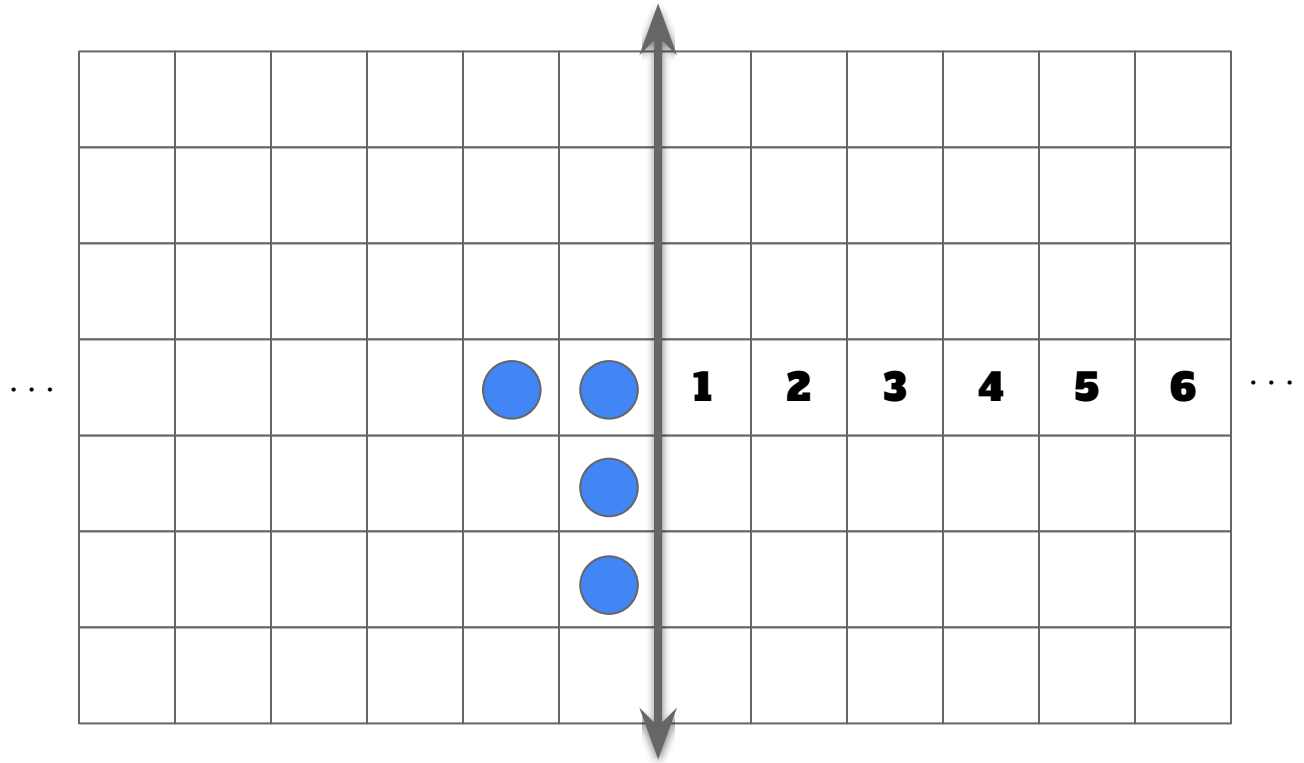
Peg Solitaire

- Goal of reducing to one peg
- Use of jump moves where jumped peg is removed
- Lots of research done on this puzzle alone
    - Conway Pagoda Function
    - NP-Completeness
    - Regular Languages

Solitaire Army Problem 1D

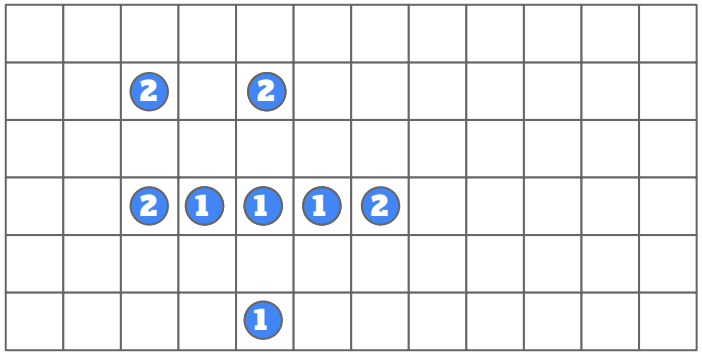


Solitaire Army Problem 2D

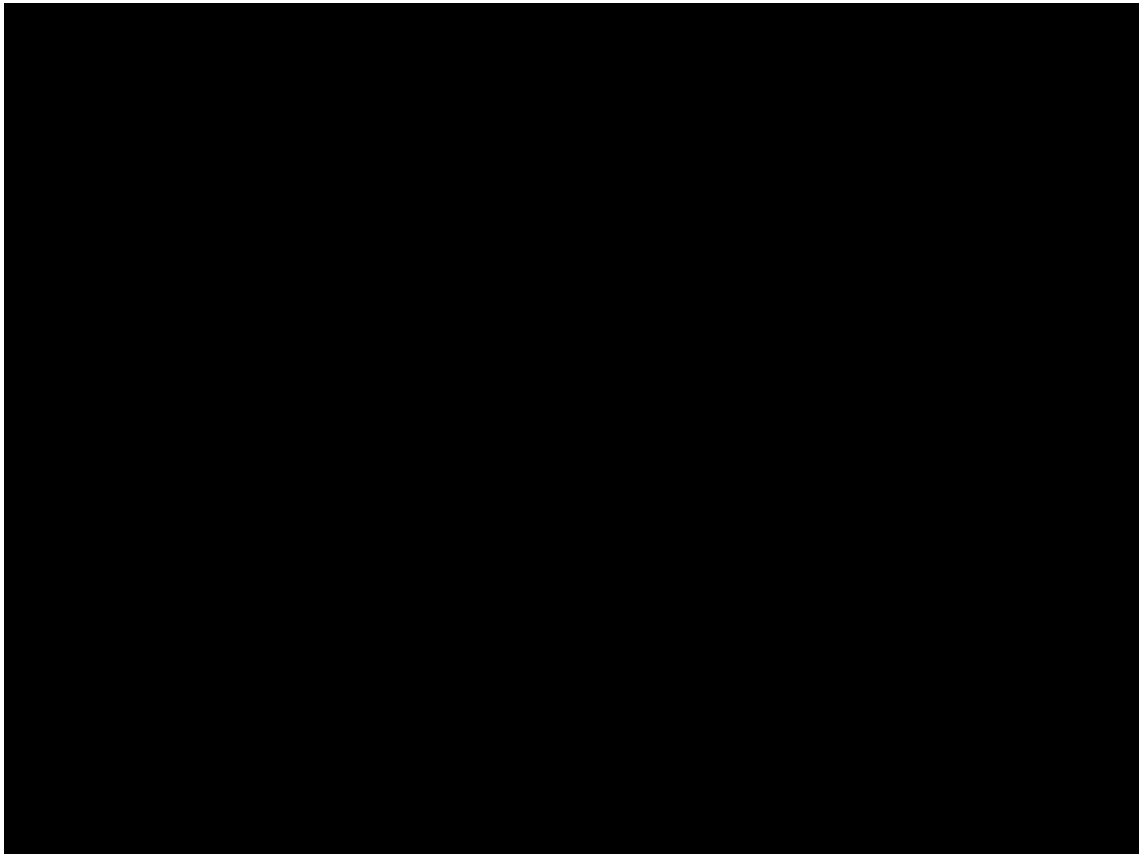


Port-and-Sweep Solitaire (PaSS) Rules

- Game is played on a 2D grid similar to Peg Solitaire
- Spaces can now hold up to 2 counters
- Two types of moves (can be played up, down, left, right).
- Port Move -------- | -2 | 0 | +1 |
- Sweep Move ---- | -1 | -1 | -1 | +2 |

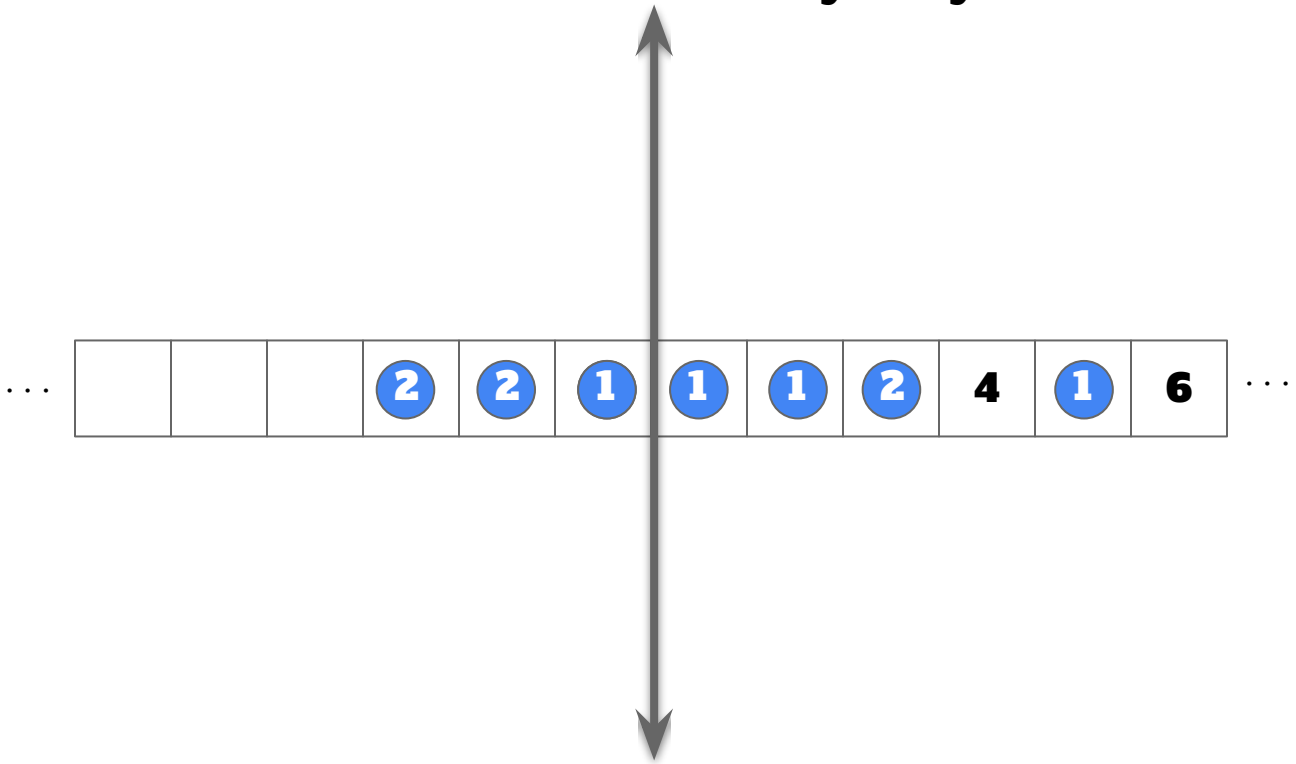


Example puzzle



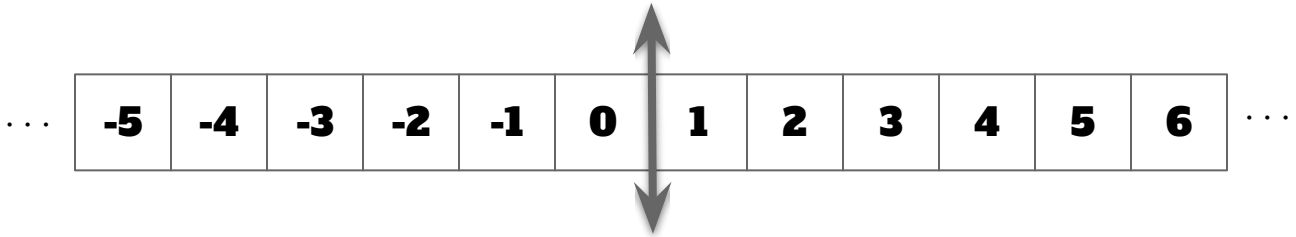
PaSS Army Problem 1D

Q: What is the furthest distance any army can advance?



Resource Counts

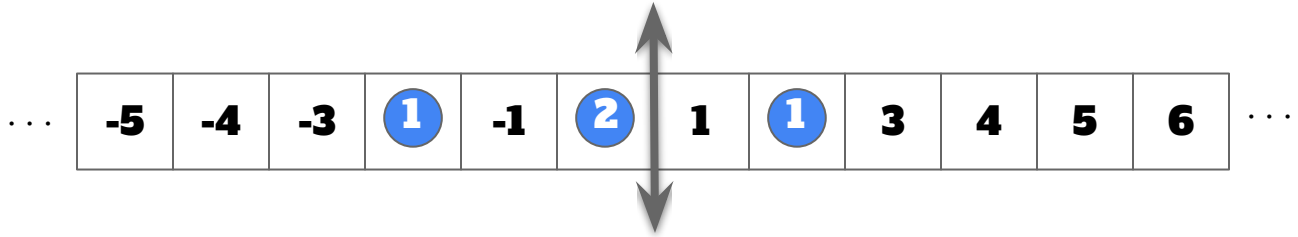
- Using the idea of a weight function, α, which gives a specific value to any board configuration



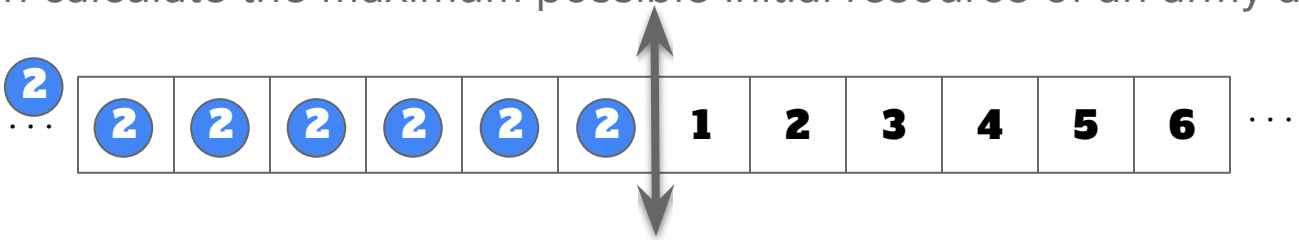
- Let $\alpha \coloneqq 1.2338$, the value of a counter at position *i* is given by α^i
- The *alpha-value* of a board is given by the sum of values of all counters
- Think of the *alpha-value* as a “resource count”
  - Non-increasing value by construction

Resource Counts

- Alpha value of the below board is given as follows



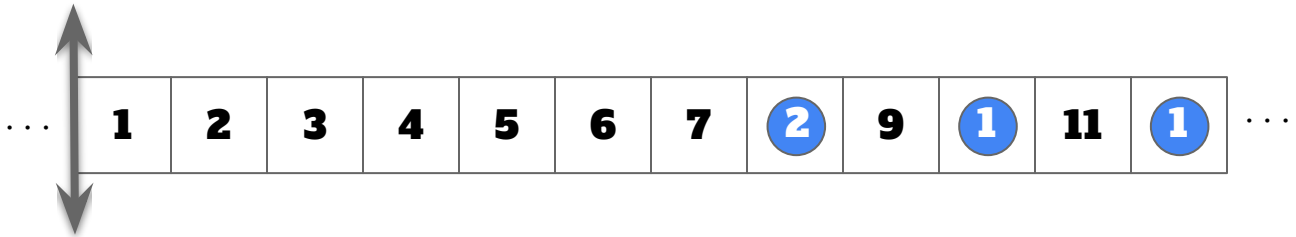
- Alpha value = $\alpha^{-2} + 2\alpha^{0} + \alpha^{2} \approx 4.18$
- We can calculate the maximum possible initial resource of an army as so



- Maximum army alpha value = $2\alpha^{0} + 2\alpha^{-1} + 2\alpha^{-2} + \ldots \approx 10.56$

Distances of 10+

- Distance of 12 requires too high of a resource count
  - Starting *alpha-value* <= 10.56 <= 12.44 ≃ α^{12}
  - Hence, army advances of 12+ are impossible
- A single counter at cell 10 implies a port from cell 8
  - Starting *alpha-value* <= 10.56 <= 10.74 ≃ $2\alpha^{8}$
  - Hence, army advances of 10+ are impossible as well



Distance of 9

- We have shown that in order to get a single counter at cell 9, these following moves have to be made at some points:
  - A rightwards port from 7 to 9
  - A rightwards port from 4 to 6
  - A rightwards port from 3 to 5

$$[\text{maximum starting resource}] - [\text{resource loss by moves}] \precsim 6.47$$

- However, $\alpha^9 \precsim 6.62$
- Hence, army advances of 9 are impossible

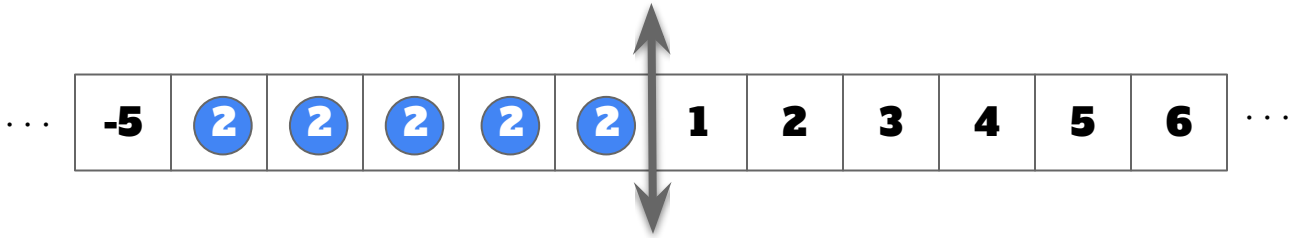
Distance of 8

- A distance of 8 is difficult to prove by deductive strategy.
- Assumption:
  - No leftwards (backwards) moves are made
  - No debris left behind
- Approach:
  - Linear algebra (linear combination of vectors)
  - Computer generation

- Able to show given above assumptions, no army can advance a distance of 8

Possible Advances

- Here is the configuration to achieve a distance of 6 with 10 counters



- How many counters do you think are needed to advance 7 spaces?
  - 12?
  - 15?
  - 20?
  - 50?

Thank you

- Professor Jacob Siehler
- Stephen Hilding Fund

More reading about game here:

Collection of puzzles to try: