One-dimensional Port-and-Sweep Solitaire Armies

Filip Belik and Ha Le
Peg Solitaire

- Goal of reducing to one peg
- Use of jump moves where jumped peg is removed
- Lots of research done on this puzzle alone
  - Conway Pagoda Function
  - NP-Completeness
  - Regular Languages
Solitaire Army Problem 1D
Solitaire Army Problem 2D
Port-and-Sweep Solitaire (PaSS) Rules

- Game is played on a 2D grid similar to Peg Solitaire
- Spaces can now hold up to 2 counters
- Two types of moves (can be played up, down, left, right).
- Port Move -------- [-2 0 +1]
- Sweep Move ---- [-1 -1 -1 +2]
Example puzzle
PaSS Army Problem 1D

**Q:** What is the furthest distance any army can advance?
Resource Counts

● Using the idea of a weight function, $\alpha$, which gives a specific value to any board configuration

\[
\alpha \approx 1.2338, \text{ the value of a counter at position } i \text{ is given by } \alpha^i
\]

● The *alpha-value* of a board is given by the sum of values of all counters

● Think of the *alpha-value* as a “resource count”
  ○ Non-increasing value by construction
Resource Counts

- Alpha value of the below board is given as follows

\[ \alpha^{-2} + 2\alpha^0 + \alpha^2 \approx 4.18 \]

- Alpha value = \( \alpha^{-2} + 2\alpha^0 + \alpha^2 \approx 4.18 \)

- We can calculate the maximum possible initial resource of an army as so

\[ 2\alpha^0 + 2\alpha^{-1} + 2\alpha^{-2} + \ldots \approx 10.56 \]

- Maximum army alpha value = \( 2\alpha^0 + 2\alpha^{-1} + 2\alpha^{-2} + \ldots \approx 10.56 \)
Distances of 10+

- Distance of 12 requires too high of a resource count
  - Starting $\text{alpha-value} \leq 10.56 \leq 12.44 = \alpha^{12}$
  - Hence, army advances of 12+ are impossible

- A single counter at cell 10 implies a port from cell 8
  - Starting $\text{alpha-value} \leq 10.56 \leq 10.74 = 2\alpha^{8}$
  - Hence, army advances of 10+ are impossible as well
Distance of 9

- We have shown that in order to get a single counter at cell 9, these following moves have to be made at some points:
  - A rightwards port from 7 to 9
  - A rightwards port from 4 to 6
  - A rightwards port from 3 to 5

  \[
  \text{[maximum starting resource] - [resource loss by moves]} \approx 6.47
  \]

- However, \( \alpha^9 \approx 6.62 \)
- Hence, army advances of 9 are impossible
Distance of 8

- A distance of 8 is difficult to prove by deductive strategy.
- **Assumption:**
  - No leftwards (backwards) moves are made
  - No debris left behind
- **Approach:**
  - Linear algebra (linear combination of vectors)
  - Computer generation
- Able to show given above assumptions, no army can advance a distance of 8
Possible Advances

- Here is the configuration to achieve a distance of 6 with 10 counters

- How many counters do you think are needed to advance 7 spaces?
  - 12?
  - 15?
  - 20?
  - 50?
Thank you

- Professor Jacob Siehler
- Stephen Hilding Fund

More reading about game here:

Collection of puzzles to try: